REPORT OF THE

CO-INSURANCE COMMITTEE

TO THE

BOARD OF FIRE UNDERWRITERS OF THE PACIFIC

ON

PERCENTAGE CO-INSURANCE

AND THE

RELATIVE RATES CHARGEABLE THEREFOR

ALSO ON THE COST OF

CONFLAGRATION HAZARD OF LARGE CITIES

SAN FRANCISCO, CALIFORNIA
September, 1905.

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To the Executive Committee, Board of Fire Underwriters of the Pacific:

Gentlemen:

Your sub-Committee on the subject of Co-insurance being adopted for San Francisco beg to report:

After careful consideration of what has been written on the subject and all the data obtainable, they are of the opinion that up to the present time all the different rate reductions for co-insurance are arbitrary to a greater or less extent, and that until other statistics are utilized than heretofore, they will so continue.

As to cities and towns where sufficiently reliable data are obtainable as to losses and values, there is no reason why the value of the percentage co-insurance clause should not be ascertained within a small degree of exactness. Those who have given or who will give careful consideration to the question will find that nothing approaching facts regarding value of co-insurance can be learned without *loss to value of property—that is the foundation stone;* it has been claimed by some that loss to insurance is good enough, especially as any error on such a basis of calculation would result in favor of the insurance companies, but it is not wise to let non-co-operating companies have so wide a margin in their favor, and as the amount of insurance carried is a movable quantity at the will of the assured, it will readily be seen that *facts* cannot be learned therefrom; advocates of this plan argue that values of property as learned from proofs of loss are not altogether reliable; that as to individual losses is occasionally likely to be the case, but in a large number of losses under- and over-statements of values will adjust themselves very closely, and after everything that may be said against the reliability of such values of property, no better method of obtaining such values has been suggested.

Your Committee are ambitious enough to wish they could settle the value of co-insurance over the entire United States, but their opportunities of getting statistical information are too limited, having regard to the area under their jurisdiction, but they are not without strong hope that this report may pave the way for Eastern organizations to co-operate on similar lines and compile the data gathered so as to make an average that could be applied over an important part of the United States. One table would not answer because in the opinion of your Committee the value of co-insurance fluctuates according to construction, climatic conditions, water supply, fire department, etc. It would, therefore, be necessary to divide cities and towns into three or more groups and have loss to value and other data secured as to a sufficient number of each group to form a fair average of the whole of such group. All towns of inferior construction, those with inadequate water supply or fire department, should be thrown out, co-insurance being of little value in such towns, and it is of no value in the isolated risk unless under superior fire protection or sprinkler equipment.

The question of mandatory co-insurance, eighty per cent co-insurance only, or leaving the percentage open and fixing rate accordingly was considered and having regard to the objection of some people to compulsory percentage of co-insurance and the enactment of anti-co-insurance laws in some States, it was deemed expedient to recommend the open percentage method in order that *the assured can select for himself what he wishes to purchase and companies can charge accordingly.*

The only available statistics of loss to value obtainable by this Committee are those relating to San Francisco fires contained in proofs of loss in various offices. Of course, similar figures could be obtained as to a number of other cities on the Coast, but in the opinion

of the Committee such other cities would belong to a different class to that in which San Francisco should be placed. It was decided to employ Mr. Albert W. Whitney, Professor of Mathematics of the University of California, and almost without exception all the leading companies gave him the opportunity of examining their San Francisco proofs of loss for the five years 1899 to 1903, from which the record of 5642 fires was tabulated. From these *facts* under the instructions of the Committee, Mr. Whitney made certain classifications and calculations as shown in his report attached hereto.

Brick special hazards were not taken into account, as the number of important fires is too limited to be of material value. Brick dwellings are not separately dealt with as there are not enough of them in San Francisco to form a class by themselves.

We are aware that the Universal Mercantile Schedule adopts fifty per cent co-insurance as the basis from which to increase or reduce rates, but as there does not appear to be any evidence that tabulation of facts demonstrated that fifty per cent was the average insurance carried on which companies have been making their experience without co-insurance, the Committee can see no reason for recommending any figure as a base line for percentage co-insurance rates, other than the actual experience as to average insurance to value heretofore carried, and we are not experimenting in doing so because the same results as to profit and loss heretofore obtained, or a little better, can be looked for in the future, provided we adhere to past experience as to per cent of insurance carried.

The relative or percentage co-insurance rate for San Francisco based on the five years' experience shown in Table 17 of Mr. Whitney's report should produce the same underwriting results as in past years, with a probable improvement in years of general business depression

(for reasons hereafter stated) by using present rates as a basis to calculate the percentage co-insurance rates.

An important point to be considered is that the five years' losses in San Francisco on which results are figured do not contain any conflagration losses, so that the question of a loading being necessary should be considered. As stated heretofore, our opinion is that the value of co-insurance has to be grouped as to three or more classes of cities and towns; therefore, conflagration hazards must be similarly dealt with, and taking (for the purpose of illustrating) as Class 1, cities showing over 200,000 population, as per 1900 census, we find nineteen such cities, and the census returns of those cities for the last six decades are 1,591,588 for 1850, 2,926,732 for 1860, 4,159,425 for 1870, 5,586,268 for 1880, 7,858,595 for 1890, and 11,795,809 for 1900. This would make a mean average annual population of 5,444,943. The premium income (for 1900) of eleven of these cities (having a population in 1900 of 8,086,649) sufficiently distinctive and remote from one another to form a good average of the whole, shows the average annual premium per capita to be $3.77. Of course, the average rate has varied during the different periods, but not sufficiently to materially affect final results.

An arbitrary period had to be taken for figuring conflagration losses and it was decided to take a period of fifty years. In these nineteen cities during the past fifty years we have had serious conflagrations in Chicago, Boston and Baltimore, causing an insurance loss of $180,116,620. The fifty years' premium income based on $3.77, the 1900 per capita, being $1,026,371,755, and the fifty years' conflagration loss to insurance companies on such premium income being $180,116,620, we find 17.55 per cent of premium is the cost of conflagration losses in cities at the present time having a population of over 200,000, and dividing the average annual con-

flagration loss by the average annual population, we find the conflagration loss to be $0.66 (sixty-six cents) annually per capita.

The Committee realize that the question of conflagration hazard was not referred to them, but it is so closely allied to the co-insurance question that attention had to be called to it in this report.

Before deciding, however, that any loading is needed to present rates for conflagration hazard, it would be necessary to ascertain the insurance losses in the nineteen cities referred to. The losses, less the conflagrations named, for the past five years or any other period, should be secured and the average annual per capita loss ascertained, add the annual conflagration loss of $0.66 per capita, then compare the result with the average annual per capita premium paid for the corresponding years, and it would then be shown whether rates needed grading up or down, or whether present rates are fair to all interests involved. This method of calculation could be applied to any one city or group of cities.

Another point to be considered is that with reduced rates on account of co-insurance, the public would be apt, especially in the better class of risks, to carry a larger amount of insurance, and that under such conditions a larger percentage of insurance to value would be involved in conflagration losses.

The figures as to conflagration losses and premiums in large cities were obtained through the courtesy of General Agent Miller of the National Board of Fire Underwriters.

Conflagrations at Seattle, Spokane, Jacksonville, Waterbury, Paterson, Rochester, etc., are not taken into account, as they would figure with the premium income and population in such group that they might be placed.

Your Committee are of the opinion that the public having the option of purchasing any percentage of in-

surance to value they wished would be disarmed from making any valid objection to co-insurance, and the companies would be protected from adverse selection being made against them by having to carry a high percentage of value on poor risks and a small percentage on good risks. With the percentage co-insurance clause and rates adjusted accordingly, the business of fire insurance will take care of itself much more evenly than at present.

It is well known that in years of general depression in business the loss ratio to premium income increases as a rule; is not that accounted for by the fact that property owners studying economy often reduce the amount of insurance carried, and the adverse experience of insurance companies in such years can with little doubt (if any), in the Committee's opinion, be attributed to the falling off in contributing insurance to partial losses. In other words, the adverse selection by the insured reducing the average amount of insurance carried to value increases loss ratio and turns profit into loss. This is a much more reasonable solution of the unprofitable results to insurance companies during years of business depression than the easier and oft-reiterated cry of increased moral hazard.

Repeating our hope that this report may pave the way for Eastern associations to take this subject up on a similar or better basis (if ascertainable) and reduce to a science the subject of percentage co-insurance and relative rates to be charged therefor.

Respectfully submitted,

C. F. Mullins,
Chairman.

Arthur M. Brown,
R. W. Osborn,
B. J. Smith,
V. Carus Driffield,
Committee.

Mr. C. F. Mullins,

Chairman of the Co-insurance Committee of the Board of Fire Underwriters of the Pacific,

Dear Sir:—I have the honor of presenting herewith, as requested by your committee, a report based on fire statistics of the city of San Francisco for the years 1899-1903, inclusive.

The primary object of this investigation has been the determination for a number of classes of the relative rates for co-insurance, the secondary object has been the putting on record of the elements involved in the scientific determination of fire insurance rates and of a plan for the calculations necessary thereto. I am,

Yours very truly,

Albert W. Whitney.

Berkeley, California, August, 1905.

I.—Introduction.

A determination of the rates for co-insurance involves, first, a thorough analysis of the structure of a rate, second, the gathering of the necessary statistics and, third, the application of the theory to the facts.

I say, first, a thorough analysis of the rate. As a matter of fact, co-insurance rates are nothing but the real rates guarded by the co-insurance clause against adverse selection, namely, the selection by the insured of less insurance than the rate was designed for.

In order to understand what, for lack of a better name, I have called the real rates, it is necessary to have clearly in mind the elements that make the fire insurance problem; but in order to understand the fire insurance problem, I propose, for the sake of the added clearness that comes from comparison, to measure it against the life insurance problem whose elements are more easily grasped.

If a man wished to buy an insurance of $1,000 for his whole life and if the element of interest were to be neglected the price of the insurance would be $1,000, for he is sure, sooner or later, to die. The time of his death is immaterial when the element of interest is left out of account, but when this is admitted the time of his death is a matter of considerable importance. If it were known positively that he would live exactly twenty years and if an interest rate of five per cent prevailed the insurance could be sold to him for the present value of $1,000 twenty years hence, or about $377. But there would be no reason for calling this insurance; it would be nothing more than an ordinary banking transaction.

But as a matter of fact the time of his death is uncertain, and it is this element of uncertainty that brings the transaction into the field of insurance. Yet all must not be uncertainty or there will be no basis for an agreement. In reality we may assume that we know four things; first,

a safe rate of interest to count upon; second, the age of the applicant; third, that he is sound physically and has a good environment; fourth, that a large number of men of his age and of an average physical condition and environment who have in the past been under observation have experienced a certain recorded mortality. We may treat this man then as though he were one of such a group. We may thereupon compute, taking account of interest, the present value of the sum needed to meet the death claims among this group as year by year they mature; this sum assessed equally among the insured is the net single premium. As a matter of fact life insurance is usually paid for in yearly installments, but as there is no analogue to this in fire insurance practice, we need not follow it out.

I have supposed the insurance to be for the whole of life; this eliminates the question of whether or not the claim will mature, and makes it a question only of when it will mature.

This fact that death is sure to occur, but that a loss by fire is not sure to occur, has been asserted to mark a vital distinction between these two types of insurance. This is wrong, however, for the certainty of death does not set any characteristic mark upon life insurance, and, as a matter of fact, term insurance in one form or another is a large part of the business of a life insurance company.

Term insurance, however, yields a problem even if interest is neglected. If a man of 35 wishes to insure his life for $1,000 during only the next five years, we may go to the mortality table and consider the 81,822 persons alive at the age of 35; we shall find that of this number 3,716 die during the next five years. If each of this group of persons were insured, the death claims would amount to $3,716,000. Neglecting interest and assessing this equally among the 81,822 persons insured would give a net single premium of about $45. If interest is taken

into account the problem is essentially the same as the problem of whole-life insurance that we have already discussed.

We have assumed that we are dealing with persons of a fairly definite type and standard of civilization, for instance, white persons living in the northern part of the United States. If we wish to transact an insurance business among, for instance, the natives of India, the problem will be the same except that we shall find a mortality experience peculiar to the class.

This is the form of a mortality table:—

TABLE 1.—THE AMERICAN EXPERIENCE TABLE.

x	d_x
10	749
11	746
12	743
"	"
"	"
"	"
"	"
"	"
"	"
"	"
"	"
93	58
94	18
95	3
Total,	100,000

The column headed x refers to the number of completed years, the column headed d_x refers to the number dying during the following year. The sum of the numbers in the second column is 100,000; that is, 100,000 persons began the 11th year together; of these, the table says, 749 died during that year, 746 during the 12th year, and so on. Among the natives of India we should obtain a different set of numbers.

We may now see clearly the elements that go to make up a life insurance rate; they are two, the law of mortal-

ity and the rate of interest. The rate then will depend, first, upon the law of mortality, which will vary with the class, second, upon the age of the insured at the time at which the insurance is effected, third, upon the rate of interest, and, fourth, of course, upon the particular form of insurance desired.

Now let us make a corresponding analysis of fire insurance conditions. In the first place fire insurance is written for such short terms that the element of interest enters in such a simple way as to be negligible in making the rate. We therefore sweep away at once what is the main factor in the structure of a life insurance rate.

The mortality among lives changes with the age. This is not true, however, among fire risks to any great extent, that is, the age of a building is not a very important element in fixing the rate, at least it need not and indeed cannot be taken account of in any such systematic way as in life insurance. This, together with the fact that fire insurance is written for such short terms, eliminates this element from the problem.

We see, therefore, that the two factors of life insurance rating, the rate of interest and the mortality in terms of age, are almost lacking in the fire insurance problem, at least they do not require a systematic treatment.

What then are the elements of the fire insurance problem? In the first place and most important of all, the element of class. Just as the rate for a man with an hereditary tendency to an organic disease or for a stoker or for an inhabitant of a tropical country should be higher than the rate for a healthy life in a healthful environment, so the rate for a saw-mill or a frame building without fire protection should be higher than for a fire-proof office building. In fire insurance the class is more important than any other element in making the rate, while in life insurance it has been, until recently,

of very little importance, for almost all life insurance has been effected in a single class, that of standard lives of white persons in non-tropical latitudes.

We now come, however, to the element that really differentiates fire insurance from life insurance. In life insurance there is no such thing as partial loss (the analogy, if any, may be sought in accident insurance); when a man dies the full face-value of the policy is drawn upon. But when a building or a stock of goods burns, it seldom burns completely; the amount of damage is an exceedingly important factor in the problem, for it determines what part of the face of the policy will have to be paid. This is the analogue to the element of mortality among lives, but instead of the time element we have the quantity element. In life insurance the mortality question is *when*, in fire insurance it is *how much*. The mortality table for life insurance gives *how many* in terms of *when*, the mortality table for fire insurance gives *how many* in terms of *how much*. This for instance is the mortality table for frame business buildings in San Francisco:

*TABLE 2.—TABLE OF PARTIAL LOSS FOR THE CLASS OF FRAME BUSINESS BUILDINGS.

x	m_x
0	8293
1	576
2	326
3	215
4	139
5	97
6	69
7	49
8	42
9	194
Total	10,000

* It is not to be inferred that this table gives the actual number of fires that have occurred in some particular time; the numbers are only relative, 10,000 having been chosen for convenience.

The column headed x refers to the number of tenths of sound value next lower than the amount destroyed, and the column headed m_x refers to the number of risks, among 10,000 losses altogether, that experience this particular range of loss. That is, out of 10,000 buildings damaged by fire 8,293 may be expected to suffer a loss of less than 1/10 of the value, 576 a loss of more than 1/10 and less than 2/10 of the value, and so on. The analogy of this to a mortality table in life insurance is, I think, obvious.

Now just as in life insurance there are different mortality tables for different classes, so in fire insurance there are different mortality tables for different classes (see Table 12). Inside of a single class then the elements of the life insurance problem are the rate of interest and the law of mortality and the problem to be solved is: with a given rate of interest and at a given age, what is the rate for an insurance?

Inside of a single class the element of the fire insurance problem is the law of mortality, or as I shall henceforth call it, the law of partial loss, and the problem to be solved is: with a given ratio of insurance to value, what is the rate for an insurance?

As a matter of fact when I say that this is the problem of fire insurance I am not stating the problem of finding the ordinary rate, but the co-insurance rate. The ordinary rate takes no account of the ratio of insurance to value, that is, a man pays the same rate for $5,000 of insurance whether it is written on a building worth $5,500 or on one worth $10,000. But manifestly the expected loss to the company is much greater in the second case than in the first; in the second case it will take only a 50 per cent loss to exhaust the insurance, in the first case it will take more than a 90 per cent loss to exhaust it. Now in the case of frame business buildings, according to our table, there are 451 chances of a 50 per cent loss to 194 chances of a 90 per cent loss.

The co-insurance rates are special averages, the ordinary rate is a general average in which the man who insures for a small amount, that is, a small ratio of insurance to value, gets his insurance too cheaply, he who insures for a large amount pays for his insurance too dearly.

The analogous rate in life insurance would be obtained by neglecting the element of age; then a young man would pay too much for his insurance in order that an old man might pay too little. Such a rate would be obtained by dividing the total amount of death claims by the total amount of insurance in force, just as in fire insurance the rate is actually obtained by dividing the total insurance loss by the insurance in force.

The weakness of a system of this kind in life insurance is this, that it leads to adverse selection; young men will not insure, therefore the mortality increases, the rate increases, the system is unstable. Hence life companies are forced to take account of the element of age; witness the experience of friendly societies.

We might expect adverse selection in fire insurance; it would consist in a refusal to carry much insurance, that is, a large ratio of insurance to value. That it does not, as a matter of fact, operate to a greater extent is due to several causes, one of which is that the insurance is often needed at any reasonable price, especially as collateral security for loans.

However, just as it is manifestly fairer and better that a man of age 25 should be rated according to the hazard at his age rather than be forced to help make up the deficit caused by a too small rate for a man of 55, so it is manifestly fairer and better that a man who wishes to insure for 90 per cent of the value of his property should be given a rate to meet the hazard rather than be forced to help make up the deficit caused by the under-rating of a man who carries 30 per cent of insurance to value.

I hardly need discuss the practical difficulties in the way of this. One of them arises from the expense of ascertaining sound values. For the company to determine sound values accurately at the time of effecting the insurance would be practically out of the question, and in the case of stocks of merchandise which are continually changing would be ineffective.

The co-insurance clause is an agreement on the part of the insured to maintain a specified ratio of insurance to value. He will maintain this in insurance companies presumably, but in case he fails so to do he shall by the agreement be regarded as himself a co-insurer for the balance. He thereby becomes jointly responsible with the other insurers, each for his share of the loss. This agreement places upon the insured the responsibility for the ascertaining of sound values and with entire fairness for he should have a sufficiently accurate knowledge of his own affairs to obtain this information easily and to order his business with this agreement in mind.

To fix the proper rate for a fire insurance it is as necessary to have this information as to ratio of insurance to value as to know the age of an applicant for life insurance. The responsibility for stating his age correctly is placed upon the insured with a penalty if he fails to do so; the co-insurance clause puts upon the insured the responsibility for keeping the ratio of insurance to value at a specified figure with a penalty if he fails to do so of having to act himself as insurer for the balance.

Or again: when a man buys \$8,000 worth of insurance and pays for it at an 80 per cent co-insurance rate it is equivalent to the admission that his insurance protects just \$10,000 worth of property (of course only partially protects, namely, up to 80 per cent of its value). The fact that he buys protection for just this

amount of property is an integral part of the transaction. If when a loss occurs the sound value of the property is greater than $10,000 it is quite obvious that the excess value is in this sense unprotected, or, if you like, is insured by himself for 80 per cent of its value. In case the damage is less than 80 per cent the insured bears of the loss only his part as a co-insurer; if the loss is greater than 80 per cent he loses not only as a co-insurer, but also because he has bought only incomplete protection.

The rates for co-insurance then are nothing but rates that take into account ratio of insurance to value. The necessary and sufficient data for their determination for a particular class are contained in what I have called the table of partial loss.

The mathematical statement of the method actually used in computing these rates will be reserved till later, for it is not necessary to a general understanding of the subject. I shall defer also the explanation of the method of obtaining this table of partial loss from the office statistics.

II.—The Co-Insurance Problem Treated Arithmetically.

As an example, let us consider the 60 per cent co-insurance* rate on frame business buildings. The table of partial loss has already been given (Table 2). As a rough approximation we might call the average loss under 10 per cent, 5 per cent, the average loss between 10 and 20 per cent, 15 per cent, and so on, but as a matter of fact it is worth while to examine our statistics closely enough to determine more accurate averages. The results are as follows:

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* Whenever *rate* is referred to in this report, *net rate* or fire cost is meant. The office rate is obtained from this simply by loading.

TABLE 3.—AVERAGE PERCENTAGE OF PROPERTY LOSS TO SOUND VALUE.

1.8	per cent.	among	losses	between	0	per cent.	and	10	per cent.
14.2	"	"	"	"	10	"	"	20	"
24.5	"	"	"	"	20	"	"	30	"
34.7	"	"	"	"	30	"	"	40	"
44.8	"	"	"	"	40	"	"	50	"
54.9	"	"	"	"	50	"	"	60	"
65	"	"	"	"	60	"	"	70	"
75	"	"	"	"	70	"	"	80	"
85	"	"	"	"	80	"	"	90	"
99.5	"	"	"	"	90	"	"	100	"

Let us now find the insurance loss on 10,000 claims, supposing that the sound value of each risk is $100 and that on each risk an insurance of 60 per cent of the value, that is, $60, is carried. But before we do this let us find the property loss. This will evidently be made up as follows:

TABLE 4.—THE PROPERTY LOSS; SOUND VALUE OF EACH RISK $100; 10,000 CLAIMS.

8293	losses of	$ 1.80	each	$14,927 40
576	"	14.20	"	8,179 20
326	"	24.50	"	7,987 00
215	"	34.70	"	7,460 50
139	"	44.80	"	6,227 20
97	"	54.90	"	5,325 30
69	"	65.00	"	4,485 00
49	"	75.00	"	3,675 00
42	"	85.00	"	3,570 00
194	"	99.50	"	19,303 00
	Entire property loss			$81,139 60

Now the insurance loss; since there is an insurance of $60 on each risk, any loss under $60 will be paid in full, but for losses over $60 only $60 on each. The insurance loss will, therefore, be:

TABLE 5.—THE INSURANCE LOSS; SOUND VALUE OF EACH RISK $100; INSURANCE $60; 10,000 CLAIMS.

8293	losses of	$ 1.80	each	$14,927 40
576	"	14.20	"	8,179 20
326	"	24.50	"	7,987 00
215	"	34.70	"	7,460 50
139	"	44.80	"	6,227 20
97	"	54.90	"	5,325 30
69	"	60.00	"	4,140 00
49	"	60.00	"	2,940 00
42	"	60.00	"	2,520 00
194	"	60.00	"	11,640 00
	Insurance loss			$71,346 60

Now if we divide this insurance loss by the number of risks of this kind among which these 10,000 losses have occurred, we shall obtain the average insurance loss per risk. This will be the average insurance loss per risk for $60 of insurance; the average insurance loss per risk per dollar of insurance will be had by dividing this by 60. This will be then the net rate or fire cost per dollar of insurance when the insurance carried is 60 per cent of the sound value. But as a matter of fact the number of risks upon which these 10,000 losses have occurred was not obtainable from the statistics at hand and therefore it has been impossible to determine the actual rate.

But relative rates are easily enough determined; for just as the insurance loss has been determined for a ratio of insurance to value of 60 per cent, so the insurance loss may be determined for any percentage of insurance to value. The results are as follows:

TABLE 6.—INSURANCE LOSSES FOR VARIOUS RATIOS OF INSURANCE TO VALUE; SOUND VALUE OF EACH RISK $100; 10,000 CLAIMS.

RATIO OF INSURANCE TO VALUE.	THE INSURANCE LOSS.
10 per cent	$31,997 40
20 "	45,726 60
30 "	55,243 60
40 "	62,154 10
50 "	67,331 30
60 "	71,346 60
70 "	74,541 60
80 "	77,146 60
90 "	79,296 60
100 "	81,139 60

With full insurance the insurance loss is equal to the property loss as it should be.

Now just as we proposed to obtain the 60 per cent rate by dividing first by the number of risks and then by 60 so we might propose to obtain the 10 per cent rate by dividing by the number of risks and then by 10, and so on. Now since the unknown element, the number of risks, enters into all the rates in the same way, in forming the relative rates it may be neglected and we may divide the insurance losses in Table 6 simply by the corresponding amounts of insurance carried. This gives:

TABLE 7.—RELATIVE RATES FOR VARIOUS RATIOS OF INSURANCE TO VALUE.

RATIO OF INSURANCE TO VALUE.	RELATIVE RATES.
10 per cent	3200
20 "	2286
30 "	1841
40 "	1554
50 "	1347
60 "	1189
70 "	1065
80 "	964
90 "	881
100 "	811

From Table 6 we have by subtraction of successive insurance losses:

TABLE 8.—COST TO THE COMPANY OF CARRYING SUCCESSIVE TENTHS OF INSURANCE TO VALUE; SOUND VALUE OF EACH RISK $100; 10,000 CLAIMS.

The 1st tenth	$31,997 40
" 2d "	13,729 20
" 3d "	9,517 00
" 4th "	6,910 50
" 5th "	5,177 20
" 6th "	4,015 30
" 7th "	3,195 00
" 8th "	2,605 00
" 9th "	2,150 00
" 10th "	1,843 00

It is hardly necesary to point out how severely the first few tenths of insurance carried tax the company in comparison with the later tenths. The cost to the company of carrying an insurance of 20 per cent of the value is more than half the cost of carrying full insurance (see Table 6). The rate for 10 per cent insurance is four times the rate for full insurance (see Table 7).

The question that now comes up to be answered is this: what relation has the ordinary rate to the co-insurance rates? This cannot be answered without additional statistical information. The information that we need is this: how much insurance do people buy in general? If on the average they insure their buildings for 20 to 30 per cent of the value the rate will be high, if on the other hand they insure well up, on the average say for 80 per cent of the value, the rate will be low. Roughly we might say that the ordinary rate will be the same as the co-insurance rate for the average ratio of insurance to value. This, however, in general will not be exactly true and we have at hand data that will allow us to make a closer determination. The same statistics that yield the table of partial loss yield also data regarding amount of insurance carried.

Out of 127 risks under observation in the class of frame business buildings under discussion, two carried between 20 and 30 per cent of insurance, five carried between 30 and 40 per cent, or in tabular form:

*TABLE 9.—THE DISTRIBUTION OF RISKS AS REGARDS RATIO OF INSURANCE TO VALUE.

RATIO OF INSURANCE TO VALUE.	NUMBER OF RISKS.
Between 0 per cent and 10 per cent	0
" 10 " " 20 "	0
" 20 " " 30 "	2
" 30 " " 40 "	5
" 40 " " 50 "	14
" 50 " " 60 "	14
" 60 " " 70 "	29
" 70 " " 80 "	21
" 80 " " 90 "	22
Over 90 per cent	20
Total	127

* I shall refer to this as the table or law of insurance to value.

The average ratio of insurance to value taken from the actual figures was 70.77 per cent.

If we were to divide each of the insurance losses in Table 6 by the number of risks we should obtain the average insurance losses per risk, but as I have said we have no information as to the number of risks. However, if we divide each of these losses by the number of risks upon which loss occurred, that is 10,000, we should obtain the average insurance loss per claim. For instance the average insurance loss per claim when 60 per cent of insurance to value is carried is $7.13466; when 70 per cent is carried is $7.45416, and for risks that range in ratio of insurance to value from 60 to 70 per cent as do the 29 in Table 9 we may with sufficient accuracy take for the average insurance loss per claim the average of the 60 and the 70 per cent values or $7.294 and call this the average insurance loss per claim when 65 per cent of insurance to value is carried, and so for the other intervals.

These intermediate values of the average insurance loss per claim are:

Table 10.—The Average Insurance Loss per Claim for Various Intermediate Values of the Ratio of Insurance to Sound Value.

RATIO OF INSURANCE TO VALUE.	AVERAGE INSURANCE LOSS PER CLAIM.
5 per cent	$1.600
15 "	3.886
25 "	5.048
35 "	5.869
45 "	6.474
55 "	6.934
65 "	7.295
75 "	7.584
85 "	7.822
95 "	8.022

Now then, assuming that the experience in Table 9 may be taken to be typical, 2/127 of 10,000 claims, on which the insurance averages 25 per cent of the value, will have an average insurance loss per claim of $5.048, or altogether $795.00; 5/127 of 10,000 claims, on which the insurance averages 35 per cent of the value, will have an average insurance loss per claim of $5.869, or altogether $2311.00; and so on, or in tabular form:

Table 11.—The Actual Insurance Loss on 10,000 Claims; Sound Value of Each Risk $100; Amount of Insurance Carried as in Table 9.

Average ratio of insurance to value.	Actual insurance loss.
5 per cent	
15 "	
25 "	$ 795 00
35 "	2,311 00
45 "	7,137 00
55 "	7,644 00
65 "	16,656 00
75 "	12,540 00
85 "	13,550 00
95 "	12,633 00
Actual insurance loss on 10,000 claims	$73,266 00

Dividing this by 10,000 we obtain $7.3266 as the actual average insurance loss per claim; I say *actual* since this is based upon figures as to insurance actually sold.

$7.3266 then is the actual average insurance loss per claim, but there is an average insurance per claim of 70.77 per cent of the value, or 70.77 dollars. The rate then per dollar of insurance actually in force will be $7.3266 divided by 70.77, or .1035. This is the rate per dollar of insurance actually in force on risks that have become claims. If this were multiplied by the ratio of the number of claims to the number of risks it would

be the burning ratio or net rate. The corresponding rate (see Table 7) with 70 per cent co-insurrance is .1065, with 80 per cent co-insurance is .0964; this actual rate of .1035 lies between these two and by interpolation it is found to agree with the rate for 73 per cent co-insurance. The corresponding problem in life insurance would be to determine the age for which the rate would be the same as the rate irrespective of age got by dividing the total amount of death claims by the total amount of insurance in force.

If then the rate in use has been properly obtained, that is, if it is the true rate, then if it is taken for the 73 per cent co-insurance rate and the other co-insurance rates are taken in proper ratios to this as determined by Table 7, the business will produce with these rates an amount sufficient to meet the expected loss just as well as with the old single rate; in fact even if the old rate is not the true rate but if it is, for instance, too large, the business conducted with the co-insurance rates will continue to produce the same income as with the old single rate, provided the law of insurance to value remains the same.

A very important thing to observe is that the co-insurance rates are entirely independent of the law of insurance to value; that was introduced only when we came to a consideration of the ordinary rate. As a matter of fact the tendency would be with a change to co-insurance rates for the average amount of insurance carried to increase. The co-insurance rates, however, would still continue perfectly to produce the income requisite to meet the expected loss. The expected loss would now be larger but it would not increase as fast as the amount of insurance in force so that the ratio of the two or the ordinary rate would fall. As the profit is proportional to the expected loss the ratio of profit to insurance in force will be proportional to the ordinary rate; a

change then to co-insurance rates would be likely to be followed by an increase in the profit, but not as great an increase as that in the insurance in force.

While then the co-insurance rates have the great advantage of being independent of the law of insurance to value so that in any case the business will take care of itself so long as the law of partial loss does not change, the ordinary rate on the other hand depends very decidedly on the law of insurance to value so that with this rate the business would not take care of itself if there were a change in this law; for instance a drop in the average ratio of insurance to value from say 70 to 50 per cent would probably convert a profitable business into a losing business.

Before proceeding to give tabular results for the several classes examined, I propose to state once more the elements involved in a rate and to suggest some terms.

The basis on which the computation is made is what I have called the table of partial loss. This will vary from one class to another. The table lacks, however, one element of being competent to give us absolute rates; it tells us how the claims are distributed as to size, but it does not tell us what proportion of risks become claims. The table, however, is competent to give us relative rates and to give us the relation between the co-insurance rates and the ordinary rate.

Let us suppose for a moment that we had this additional information. Then we should be able to compute the insurance loss for any given percentage of insurance to value, not merely per claim, but per risk, and from this per dollar of insurance. This would be the net co-insurance rate, the fire cost or, I should like to call it, the measure of the hazard. We should find on examination that this consists of two factors, one being the ratio of the number of claims to the number of risks; this can be interpreted as the probability that a fire will occur; this I

propose to call the ignition hazard. The second factor is the probability that, a fire having started, the amount at stake, namely, the amount of insurance on the risk, will be lost to the company; this I propose to call the damage hazard. The product of these two, the ignition hazard and the damage hazard, is the measure of the *hazard* or the fire cost for this particular ratio of insurance to value; it is the probability first that a fire will occur, and, second, that having occurred, there will be a loss of the insurance.*

The ignition hazard might be the same under some conditions on a stock of millinery in a brick building as on a stock of groceries, but the damage hazard in the case of the millinery would certainly be much larger than in the case of the groceries.

The ignition hazard is independent of the ratio of insurance to value, the damage hazard on the other hand depends upon this as well as upon the susceptibility to damage of the risk since it is the probability of a loss of the *insurance.*

Just as we have analyzed the co-insurance rates so we may analyze the ordinary rate. It will be found to consist in the same way of two factors, the ignition hazard and the damage hazard. The ignition hazard, since it depends only upon the class, will have exactly the same value as in the co-insurance rates; the damage hazard, which is the probability that the amount at stake, namely, the average amount of insurance per risk, will be a loss to the company, will, however, differ from the damage hazards with co-insurance.

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* It should be noted that in reality the insurance loss per risk from which the fire cost is got is a very complex thing made up as it is of the sum of a number of partial losses, some of which do and some of which do not exhaust the insurance. It however reduces to the form of a single simple expectation of losing the whole amount at stake, so that for instance in the case of 60 per cent co-insurance already discussed. the insurance loss per claim made up of separate items as in Table 5 (dividing by 10000) reduces to a single quantity $7.1346 which is the amount at stake, $60, multiplied by .1189, the damage-hazard. The fire-cost comes by multiplying this by the ignition-hazard.

The ignition hazard involves the very element that our statistics fail to give and is therefore treated as unknown in this report. The damage hazards, on the other hand, we can compute completely. As the practical problem of relative rates for co-insurance involves only the damage hazards it is therefore entirely soluble.

III.—Results for Eight Classes in Tabular Form.

It was originally intended to embrace in the investigation ten classes, first, frame business buildings; second, contents of the same; third, brick business buildings; fourth, contents of the same; fifth, dwellings, (frame); sixth, contents of the same; seventh, frame special hazards; eighth, contents of the same; ninth, brick special hazards; tenth, contents of the same. It was found, however, upon examination of the statistics that they were insufficient in number in the last two classes to give reliable results. The results for the other classes are given herewith in tabular form:

TABLE 12.—PARTIAL LOSS FOR VARIOUS CLASSES.

	Frame Business Buildings	Contents of Frame Business Buildings	Brick Business Buildings	Contents of Brick Business Buildings	Dwellings	Contents of Dwellings	Special Hazards (Frame)	Contents of Special Hazards (Frame)
Number of Losses between 0/10 and 1/10 of the value	8293	7172	9370	6724	8627	8360	5413	4264
" " 1/10 and 2/10 "	576	915	337	965	502	658	791	806
" " 2/10 and 3/10 "	326	425	144	580	201	284	539	668
" " 3/10 and 4/10 "	215	310	85	429	130	178	431	553
" " 4/10 and 5/10 "	139	221	42	324	93	114	360	507
" " 5/10 and 6/10 "	97	180	16	260	73	85	309	507
" " 6/10 and 7/10 "	69	147	4	214	60	64	273	530
" " 7/10 and 8/10 "	49	123	2	183	53	57	237	553
" " 8/10 and 9/10 "	42	98	0	167	48	68	209	599
" " 9/10 and 10/10 "	194	409	0	154	213	132	1438	1013
Total	10000	10000	10000	10000	10000	10000	10000	10000

TABLE 13.—INSURANCE LOSS ON 10,000 CLAIMS, EACH OF SOUND VALUE $100.

	Frame Business Buildings	Contents of Frame Business Buildings	Brick Business Buildings	Contents of Brick Business Buildings	Dwellings	Contents of Dwellings	Special Hazards (Frame)	Contents of Special Hazards (Frame)
Insurance Loss when 1/10 of the value is insured	$31,997	$ 45,493	$16,701	$ 47,553	$26,412	$35,544	$ 59,944	$ 69,299
" 2/10 "	45,727	68,466	21,046	74,716	37,230	48,128	101,622	122,387
" 3/10 "	55,244	85,258	23,184	94,636	44,835	56,386	136,833	168,281
" 4/10 "	62,154	98,495	24,224	109,672	50,846	62,423	167,248	208,136
" 5/10 "	67,131	109,126	24,645	121,007	55,762	67,030	193,708	242,691
" 6/10 "	71,347	117,778	24,784	129,461	59,860	70,656	216,823	272,176
" 7/10 "	74,542	124,813	24,824	135,571	63,300	73,546	237,028	296,476
" 8/10 "	77,147	130,498	24,834	139,696	66,175	75,831	254,683	315,361
" 9/10 "	79,297	135,078	24,834	142,071	68,545	77,491	270,108	328,486
" 10/10 "	81,140	138,964	24,834	143,380	70,355	78,613	284,488	336,792

TABLE 14.—THE DAMAGE HAZARD, OR THE AVERAGE INSURANCE LOSS PER DOLLAR OF INSURANCE PER CLAIM AND THEREFORE THE RELATIVE NET RATE.

	Frame Business Buildings	Contents of Frame Business Buildings	Brick Business Buildings	Contents of Brick Business Buildings	Dwellings	Contents of Dwellings	Special Hazards (Frame)	Contents of Special Hazards (Frame)
The Damage Hazard when 1/10 of the value is insured	.3200	.4549	.1670	.4755	.2641	.3554	.5994	.6930
" 2/10 "	.2286	.3423	.1052	.3736	.1862	.2406	.5081	.6120
" 3/10 "	.1841	.2842	.0773	.3155	.1494	.1880	.4561	.5609
" 4/10 "	.1554	.2462	.0606	.2742	.1271	.1561	.4181	.5204
" 5/10 "	.1347	.2183	.0493	.2420	.1115	.1341	.3874	.4854
" 6/10 "	.1189	.1963	.0413	.2158	.0995	.1178	.3614	.4536
" 7/10 "	.1065	.1783	.0355	.1937	.0903	.1051	.3386	.4235
" 8/10 "	.0964	.1631	.0310	.1746	.0827	.0948	.3184	.3942
" 9/10 "	.0881	.1501	.0276	.1579	.0762	.0861	.3001	.3650
" 10/10 "	.0811	.1390	.0248	.1434	.0704	.0786	.2845	.3368

TABLE 15.—RATIO OF INSURANCE TO VALUE.

	Frame Business Buildings	Contents of Frame Business Buildings	Brick Business Buildings	Contents of Brick Business Buildings	Dwellings	Contents of Dwellings	Special Hazards (Frame)	Contents of Special Hazards (Frame)
Number of Risks on which the Insurance is between 0/10 and 1/10 of the value	0	0	1	0	0	0	0	0
" " 1/10 " 2/10 "	0	5	0	2	0	1	2	7
" " 2/10 " 3/10 "	2	7	1	9	3	4	6	9
" " 3/10 " 4/10 "	5	7	7	9	7	11	3	11
" " 4/10 " 5/10 "	14	14	9	14	11	20	14	10
" " 5/10 " 6/10 "	14	18	6	22	18	25	13	13
" " 6/10 " 7/10 "	29	27	6	27	37	43	10	19
" " 7/10 " 8/10 "	21	24	10	24	46	38	8	9
" " 8/10 " 9/10 "	22	20	14	20	59	47	9	10
" " 9/10 " 10/10 "	12	15	8	28	28	5	2	2
Number of Risks on which the Insurance is equal to or greater than the value..................	8	30	2	9	26	2	7	10
Total Number of Fires on which values were given	127	167	64	164	235	196	74	100
Number of Fires on which values were not given...	440	566	229	348	1462	1239	114	117
Total Number of Fires examined	567	733	293	512	1697	1435	188	217
Percentage of cases in which sound value was given	22.4	22.8	21.8	32	13.8	12.9	39.3	46.1

Total number of fires examined in all classes..............................	5642
Total number of fires examined in all classes on which sound value was given.......	1127
Percentage of cases in which sound value was given	20

TABLE 16.

	Frame Business Buildings	Contents of Frame Business Buildings	Brick Business Buildings	Contents of Brick Business Buildings	Dwellings	Contents of Dwellings	Special Hazards (Frame)	Contents of Special Hazards (Frame)
The actual Insurance loss on 10,000 claims.	73268	120820	24333	128990	64205	71467	211099	250163
The average percentage of insurance actually in force to value.	70.77	76.77	67.42	69.66	79.31	66.84	61.95	60.67
The actual Damage hazard.	.1035	.1574	.0361	.1852	.0810	.1069	.3408	.4123
The percentage of insurance to value at which the co-insurance damage hazard is equal to the actual damage hazard, that is at which the co-insurance rate is equal to the ordinary rate.	73.	84.4	69.	74.4	82.6	68.6	69.1	73.8

TABLE 17—RELATIVE CO-INSURANCE RATES, THE 100% RATE BEING TAKEN AT THAT TENTH OF INSURANCE TO VALUE NEAREST TO THE POINT AT WHICH THE CORRESPONDING CO-INSURANCE RATE IS EQUAL TO THE ORDINARY RATE.

RATIO OF INSURANCE TO VALUE	Frame Business Buildings	Contents of Frame Business Buildings	Brick Business Buildings	Contents of Brick Business Buildings	Dwellings	Contents of Dwellings	Special Hazards (Frame)	Contents of Special Hazards (Frame)
10 per cent	300	279	470	245	319	338	177	164
20 " "	215	210	296	193	225	229	150	145
30 " "	173	174	218	163	181	179	135	132
40 " "	146	151	171	142	154	148	124	123
50 " "	126	134	139	125	135	128	114	115
60 " "	112	120	116	111	120	112	107	107
70 " "	100	109	100	100	109	100	100	100
80 " "	90.5	100	87.3	90.1	100	90.2	94.0	93.1
90 " "	82.7	92	77.7	81.5	92.1	81.9	88.6	86.2
100 " "	76.2	85.2	69.9	74	85.1	74.8	84.0	79.5

IV.—The Statistical Problem.

In the foregoing pages I have tried to show that the co-insurance problem may be solved when the laws of partial loss and of ratio of insurance to value are known, but as a matter of fact by far the most laborious part of this investigation has been the determination of these laws, even taking no account therein of the difficulty of getting the statistics.

No data in the least adequate for such an investigation as this were immediately at hand and there was no possibility of obtaining them except from proofs of loss in the separate offices. A letter addressed to the Board offices by the Chairman of the Co-insurance Committee requesting that access to proofs of loss for the years 1899-1903, inclusive, be given to myself or a representative, brought a favorable response from about ninety companies.

The work of examining this material and collecting from it the necessary data was excellently done by Mr. A. H. Mowbray, now in the Actuarial Department of the New York Life Insurance Company; Mr. Mowbray was assisted for a time by Mr. Hart Greensfelder.

The card system was used; each card represented a risk upon which a fire loss had occurred; on each card were places for the date and location of the fire and marks for avoiding and detecting the multiple recording of a risk which was insured in more than one company.

The information really desired was the class to which the risk belonged, the sound value, the amount of insurance carried and the value of the property destroyed; there were places for these data on each card.

If it had been possible to obtain information on every risk with regard to each of these four items the problem and its treatment would have been comparatively simple. Three of the items, the class, the property loss and the amount of insurance carried were indeed obtainable on practically every risk, but the fourth item, the sound value, in only about twenty per cent of the cases.

We have seen that the co-insurance problem arises entirely from the circumstance of varying ratios of loss to sound value. To have information as to the sound value is then absolutely essential for the solution of the problem and yet the situation was this, that in eighty per cent of the cases the sound value was not obtainable.

This was so discouraging as to make it seem almost impossible to succeed with the investigation. The ray of hope that came to one for an instant of being able to throw aside the eighty per cent of cases where sound value was not given and treat only the remaining cases disappeared completely as soon as one realized that the cases in which sound value is given are highly selective, they are, namely, in general, or at least largely, just those cases in which the loss has been relatively large, and they would therefore yield entirely misleading results.

The question then is: how can we introduce the element of sound value into this eighty per cent of cases in which we have only property loss and amount of insurance given?

Fortunately this is not quite so hopeless as it seems at first. If every risk were insured for just three-quarters of its value we should be able to infer the value from the amount of the insurance and therefore the ratio of loss to value, a fifty per cent ratio of loss to insurance, for example, would mean a 37½ per cent ratio of loss to value.

Now it can indeed be shown that the *average* ratio of insurance to value is somewhere near 75 per cent and it is evident that a fairly good approximate result would be obtained if we assumed that 75 per cent of insurance (or whatever the exact value of the average might be) were carried on *each* risk, relying upon the risks carrying over 75 per cent to offset those carrying less than 75 per cent. But as a matter of fact this offsetting would by no means be perfectly effective. Suppose, for instance, the ratio of insurance to value in one case to be 120 per cent (as might

easily happen on stocks of goods) and in another case to be 30 per cent, an average of 75 per cent, and suppose that the ratio of loss to insurance has been in each case 5/6; then, as a matter of fact, on one the ratio of loss to value would be 120 per cent of 5/6, that is a total loss, in the other case a loss of only 30 per cent of 5/6, or one-fourth of the value. Now if the average ratio of insurance to value, 75 per cent, were used we should obtain two losses of 75 per cent of 5/6, or 62½ per cent of the value. But from the point of view of co-insurance two losses of 62½ per cent of the value are very different from one total loss and another loss of 25 per cent. This is a case where, in forming an average, we lose the very facts that we need, that is this is a problem that needs something finer than an ordinary average.

It is to be said, however, that 75 per cent (or whatever it may more exactly be) is not only the average ratio of insurance to value but, as a matter of fact, the greater part of all risks are written for nearly this ratio and for these the use of the average ratio instead of the actual ratios would lead to sufficiently accurate conclusions. The cases in which we should be led into error are, however, of considerable importance. We should lose for one thing nearly all cases of total loss, a fully insured total loss, for instance, would count only as a 75 per cent loss. There is, however, again this to be said that in nearly all cases where there have been relatively large losses the sound values are given and we should, therefore, not be driven to the expedients that we are discussing.

However, the point is simply this, that as a matter of fact there is a better method of procedure available than that involved in assuming a uniform 75 per cent ratio of insurance to value and that the importance of the problem demands its use.

I will explain the method actually used by applying it to the class of frame business buildings. In this class our

statistics furnished records, during the five years, of 567 losses; for 127 of these, or about 22 per cent, the sound value was given. For these 127 risks for which sound value was given the table of partial loss and the table of ratio of insurance to value were as follows:

TABLE 18, PARTIAL LOSS.

x	m_x
0	71
1	17
2	12
3	6
4	4
5	4
6	2
7	1
8	2
9	8
Total,	127

TABLE 19, INSURANCE TO VALUE.

x	n_x
0	0
1	0
2	2
3	5
4	14
5	14
6	29
7	21
8	22
9	13
10	4
11	1
12	1
13	0
14	1
Total,	127

There is, furthermore, a table of ratios of loss to insurance starting out with 64 risks for which this ratio is less than 10 per cent, 14 for which the ratio is greater than 10 and less than 20 per cent, and so on. Such a table as this contains *all* the information to be had on such risks as the 440 upon which sound value is not given.

However, even these three tables do not exhibit all the information obtainable from our statistics; we exhibit, for instance, 17 losses of between 10 and 20 per cent of the value, but we do not exhibit just what per cent of insurance is carried on each of these particular losses. To exhibit our information completely we must use a double entry table such as the following:

TABLE 20.—A TABLE SHOWING THE DISTRIBUTION OF 127 RISKS CLASSIFIED AS REGARDS BOTH RATIO OF LOSS TO INSURANCE AND RATIO OF INSURANCE TO VALUE.

Percentage ratio of Insurance to Value.	Total	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100	100–110	110–120	120–130	130–140	140–150	150–160	160–170	170–180	180–190	190–200	200–210	210–220	220–230
140–150	1				1																			
130–140	0																							
120–130	1								1															
110–120	1				1																			
100–110	4	1	1				1				1													
90–100	13	10	1					1			1													
80–90	22	16	1	3	1				1															
70–80	21	11	2	2	4	1			1															
60–70	29	18	4	1	1		1			1			1			1	1							
50–60	14	1	3	1		1	1		2	1	1		1						1	1				
40–50	14	5	2	1		1		1	2	1												1		
30–40	5	1			1	1	1																1	
20–30	2	1																						1
10–20	0																							
0–10	0																							
Total 127 (Both Ways)		64	14	8	9	4	4	2	7	3	3	0	2	0	0	1	1	0	1	1	0	1	1	1

Percentage ratio of Loss to Insurance.

380 24 7 9 5 2 2 0 2 5 3 0 0 0 1

Distances measured to the right refer to ratio of loss to insurance, distances up refer to ratio of insurance to value, starting in each case at the lower left-hand corner at zero and increasing by 10 per cent intervals; the figures refer to the number of risks having at the same time the indicated values of these two quantities, for instance there are three risks on which the insurance was between 80 and 90 per cent of the value, and on which the loss was between 20 and 30 per cent of the insurance. The figures of Table 19 and the table of ratio of loss to insurance referred to are found by summing this table horizontally and vertically; for instance without regard to the ratio of loss to insurance we find 29 risks for which the ratio of insurance to value is between 60 and 70 per cent.

Now if our proposed assumption of 75 per cent ratio of insurance to value were really true we should have no such vertical distribution as this, but the figures would be massed in the eighth row.

We are able to construct such a table as this because we have sound values given. If sound values were not given our only knowledge would be that of ratio of loss to insurance, that is we should know the numbers that correspond to those that sum the columns, but we should not know them in the distributed form. If, however, we could only distribute these numbers in such manner as the numbers in the table are distributed we should be in a fair way to solve our problem, for, if we know both the ratio of insurance to value and the ratio of loss to insurance, by multiplication we obtain the ratio of loss to value, the desired information. To make this more definite, consider the actual figures. I have arranged the 440 risks for which sound value was not known, grouped according to ratio of loss to insurance, under the corresponding groups in Table 20.

Now although we know that of these 440 losses, 380, for instance, were for less than 10 per cent of the value, we

do not know anything about the ratio of the insurance to the value. Is it not reasonable, however, to suppose that the relative distribution as to ratio of insurance to value of these 380 was much the same as for the corresponding 64 for which sound value was known? If then we make this assumption, which is, in short, that the risks upon which sound value is known do not differ materially so far as their relative distribution as to ratio of insurance to value is concerned from the risks upon which sound value is not known, if we make this assumption, I say, then we may distribute these 380 losses vertically, throwing, for instance, 18/64 of them into the seventh row and so on, and so also with the other numbers 24, 7 and so on, and thus produce a table resembling the one actually obtained for the 127 risks upon which sound value was given. We should then know for each risk not only ratio of loss to insurance but ratio of insurance to value and so could obtain the desired information, namely, the ratio of loss to value.

This procedure of course would be almost prohibitive because of the labor involved and as a matter of fact it would be fruitless to follow out the accidental peculiarities that are bound to occur when the mass of statistics is so limited, but it was found that entirely satisfactory results could be obtained if instead of trying to match the relative distribution in each separate column we assumed the relative distribution in each column to be the same as the general distribution got by summing the rows horizontally, that is the distribution that we have called the law of insurance to value.

This is not a rigorously valid assumption for if risks on which the ratio of loss to insurance was large were so distributed, some would fall well up in the table where entries fail to be interpretable, for instance there could be no risk on which at the same time the ratio of loss to insurance was 150 per cent and the ratio of insurance to value was 90 per cent, for in that case the loss would be

135 per cent of the value, which is manifestly absurd; this was remedied in a somewhat arbitrary way by throwing down and back into the table the few entries that fell above the proper limits.

While this whole method of procedure I am aware sounds rather arbitrary when thus described, as a matter of fact it gave a distribution which was an enormous improvement over the crude assumption of a uniform 75 per cent ratio of insurance to value, which in itself we have recognized as not so altogether bad as not to be competent to give us fairly good results.

In order to test the effect of this method of distribution, I used it upon the 127 risks upon which sound value was given; that is although I knew the actual distribution given in the table and hence the desired information, concerning ratio of loss to value, I proceeded from the summation numbers 64, 14, 8 and so on, which would correspond to the sum total of all knowledge obtainable in the cases in which sound value was not given, I proceeded, I say, to distribute these in the manner described and thus to reconstruct the table. From the reconstructed table ratios of loss to value were obtained and hence a table of partial loss; this was compared with the table of partial loss obtained directly from the statistics. This was done for two classes and the agreement in each case was very close, so close in fact as to serve perfectly well as a smoothed expression of the law of partial loss.

This seemed thoroughly to justify the method. It was therefore used upon that portion of the statistics upon which sound values were not known and a table of partial loss for these risks was formed; this was then united with the table of partial loss obtained directly from the statistics for those risks upon which sound value was given, the two of course properly weighted, and thus a final law of partial loss was obtained.

As a matter of fact, however, there were other difficulties to be dealt with before these results could be

actually obtained. Let us suppose a table constructed by the method described; and let us suppose, for instance, five risks upon which the ratio of loss to insurance is between 20 and 30 per cent and the ratio of insurance to value between 60 and 70 per cent, then for each of these five risks the ratio of loss to value is somewhere between 12 and 21 per cent; similarly let us suppose two risks for which the ratio of loss to insurance is between 70 and 80 per cent and the ratio of insurance to value between 80 and 90 per cent, then for each of these two risks the ratio of loss to value will be between 56 and 72 per cent. Now similarly we should obtain a great array of intervals for ratios of loss to value, but instead of intervals bounded by exact numbers of tenths we should have a great variety of overlapping intervals with as many different boundaries as there are different numbers in the multiplication table. Such confusion would be hopeless.

It was necessary, therefore, to abandon classification by equal intervals and to substitute a set of intervals bounded by numbers whose products all belong to the same system. Such a system of numbers is formed by the powers of some given base. In this case the base 8/10 was selected and the intervals from $(8/10)^{20}$ to 1 were used; the risks that fell below $(8/10)^{20}$ were taken all together. This had also the advantage over the decimal system of decreasing the size of the intervals toward that end of the table at which the data were more numerous and where greater refinement of treatment was necessary.

This method of treatment proved to be entirely satisfactory except that it involved considerable work, particularly in the final passage back to decimal divisions which was necessary in order to obtain a usable table of partial loss and also in the carrying out of certain distributions or interpolations, the necessity for which had perhaps better be pointed out. A certain number of risks

are found to have a ratio of loss to value lying between $(8/10)^8$ and $(8/10)^{10}$ as, for instance, the risks whose ratio of loss to insurance lie between $(8/10)^5$ and $(8/10)^6$, and whose ratio of insurance to value lie between $(8/10)^3$ and $(8/10)^4$; a certain other number of risks are found to have a ratio of loss to value lying between $(8/10)^9$ and $(8/10)^{11}$, as, for instance, the risks whose ratio of loss to insurance lie between $(8/10)^6$ and $(8/10)^7$ and whose ratio of insurance to value lie between $(8/10)^3$ and $(8/10)^4$. These two intervals evidently overlap and a distribution is necessary among the three intervals $(8/10)^8$ to $(8/10)^9$, $(8/10)^9$ to $(8/10)^{10}$, and $(8/10)^{10}$ to $(8/10)^{11}$. The method of distribution actually used need not be discussed, as a satisfactory method would readily occur to anyone who handled the problem. In fact the whole statistical treatment is too full of detail to give in full in this report, detail, however, which would readily be supplied by anyone who undertook to work with the problem; I have attempted to give only an outline of the method employed.

It is appropriate at this point while the tedious complexity of these processes is freshly in mind to reflect that they are made necessary only by the fact that so few of the proofs of loss contain sound value. The device that I have sketched must be understood to be an expedient that was resorted to only out of necessity. The question then arises: if the co-insurance problem is worth solving is it not right that the way of the future computer should be made easier and the result more perfectly trustworthy by insisting that proofs of loss shall always contain sound value? Even if in a large number of cases the values given are only estimates, the estimates will be far better than nothing, for when dealt with in a mass inaccuracies in the estimates will largely neutralize each other.

As for this solution I may say that there has been nothing to indicate that the results are not trustworthy,

in fact I have been somewhat surprised to find how in every case where partial checks were possible, the results stood the test.

I may say also that the method that I have used is reducible to mechanical operations that can easily be run through by an ordinary computer and the difficulties are not greater than those that are common to most actuarial work.

V.—The Co-insurance Problem Treated Algebraically

Let a certain interval of time be under consideration, presumably one year.

Let us confine our attention to risks that all belong to a single class.

Let V be the sound value of each risk.

Let N be the number of risks insured.

Let L_x be the insurance loss on these N risks when each risk is insured for $x/10$ of its value. We shall later consider the case in which the risks are not all insured for the same amount.

Let l_x be the average insurance loss per risk. $l_x = L_x/N$.

Let M be the total number of risks upon which loss is incurred.

Let m_x be the number of risks on which the property loss is greater than $x/10\ V$ but less than $x+1/10\ V$, on the average (call it) $a_x/10\ V$; then $m_0 + m_1 + \ldots\ldots + m_9 = \sum_0^9 m_i = M$.

Let $\mu_x = m_x/M$.

$$\text{Then } L_x = m_0 \, a_0/10 \, V + m_1 \, a_1/10 \, V + \ldots\ldots + m_{x-1} \, a_{x-1}/10 \, V$$

$$+ x/10 \, V \left\{ m_x + m_{x+1} + \ldots m_9 \right\} = M \, V \left\{ \sum_0^{x-1} \mu_i \, a_i/10 + x/10 \sum_x^9 \mu_i \right\}.$$

Let $\sum_{0}^{x-1} \mu_i \, {}^{a_i}/_{10} + {}^{x}/_{10} \sum_{x}^{9} \mu_i = \lambda_x$; (2)

then $L_x = MV\lambda_x$, (1)

$l_x = {}^{L_x}/_{N} = {}^{M}/_{N} \; V \lambda_x$.

Let ${}^{M}/_{N} = J$; (4)

then $l_x = J \, V \, \lambda_x$. (3)

Let R_x be the average insurance loss per dollar of insurance (hence the measure of the hazard or net rate) when each risk is insured for ${}^{x}/_{10}$ V.

Let I_x be the total amount of insurance on N risks when each risk is insured for ${}^{x}/_{10}$ V;

then $I_x = N \, {}^{x}/_{10} \, V$;

$$R_x = {}^{L_x}/_{I_x} = \frac{M \; V \; \lambda_x}{N \; {}^{x}/_{10} V} = J \; {}^{\lambda_x}/_{{}^{x}/_{10}} .$$

Let ${}^{\lambda_x}/_{{}^{x}/_{10}} = \rho_x$; (6)

then $R_x = J \; \rho_x$, (5)

$l_x = J \, V \, \lambda_x = {}^{x}/_{10} \, V \, R_x$. (7)

l_x is the expectation of insurance loss per *risk* ; it is made up by (3) of two factors, J and $V\lambda_x$.

J is the probability that a given risk will become a claim; it may be called the ignition hazard. $V\lambda_x$ is the expectation of insurance loss per *claim ;* it is made up by (2) of ten elementary expectations. The product of $V\lambda_x$, the expectation per claim, by J, the probability that a given risk will become a claim, is the expectation per risk or l_x.

But l_x, instead of being looked at as the sum of a number of elementary expectations, may be thrown into the form (7) in which it is an equivalent single expectation ; the amount at stake is the insurance, ${}^{x}/_{10}$ V ; the probability of its being called out on a given risk is R_x, the measure of the hazard.

R_x in turn is made up of two factors, J, the ignition hazard, and ρ_x which may be called the damage hazard. J is the probability that a given risk will become a claim; ρ_x is the probability that, having become a claim, the insurance $^x/_{10}V$ will be called out; ρ_x is the measure of the hazard among the claims, R_x is the measure of the hazard among the risks.

Neither J nor ρ_x depend upon the sound value V. J does not depend upon the ratio of insurance to value; ρ_x however is a function both of the ratio of insurance to value, since it is the probability of insurance loss, and of the damageability of the risk as given by the μ's.

This, so far, is all under the assumption that the insurance in every case is just $^x/_{10}V$. Let us now suppose that the N risks are not all insured for the same amount.

Let n_x be the number of risks insured for more than $^x/_{10}V$ and less than $^{x+1}/_{10}V$, on the average (call it) $^{b_x}/_{10}V$; then

$$n_0 + n_1 + \ldots\ldots + n_9 = \sum_0^9 n_i = N.$$

Let $\nu_x = {}^{n_x}/_N$.

Just as l_x is the insurance loss per risk when there is an insurance of $^x/_{10}V$ so let l'_x be the insurance loss per risk when there is an insurance of $^{b_x}/_{10}V$. In general let primed symbols refer to the case where the insurance is $^{b_x}/_{10}V$. The expressions for this case are the same as those already given after a change of $^x/_{10}$ to $^{b_x}/_{10}$, for instance: $l'_x = {}^{b_x}/_{10}V\,R'_x$, the product of the amount at stake, namely the insurance, $^{b_x}/_{10}V$, and the measure of the hazard, R'_x, where $R'_x = J\rho'_x$ and $\rho'_x = {}^{\lambda'_x}/_{{}^{b_x}/_{10}}$. As a matter of fact λ'_x may with sufficient accuracy be taken to be $\frac{\lambda_x + \lambda_{x+1}}{2}$, although a closer determination might be made if it were thought desirable.

Let $\overline{L}$ be the *actual* insurance loss among N risks insured. Let us use *actual* to refer to the case where the distribution of risks as to amount of insurance carried is described by the n's, and in general let the barred symbols refer to this *actual* case.

Then $\overline{L} = n_0 l'_0 + n_1 l'_1 + \ldots\ldots + n_9 l'_9 = \sum_0^9 n_i l'_i = NJV\sum_0^9 \nu_i \lambda'_i.$

Let $\sum_0^9 \nu_i \lambda'_i = \overline{\lambda};$ (9)

then $\overline{L} = NJV\overline{\lambda} = MV\overline{\lambda},$ (8)

$\overline{l}$, the actual average insurance loss per risk, $= \overline{L}/N = JV\overline{\lambda}.$ (10)

$$\overline{I} = n_0 \, {}^{b_0}\!/_{10} V + n_1 \, {}^{b_1}\!/_{10} V + \ldots\ldots\ldots\ldots n_9 \, {}^{b_9}\!/_{10} V = NV \sum_0^9 \nu_i \, {}^{b_i}\!/_{10}.$$

$\sum_0^9 \nu_i \, {}^{b_i}\!/_{10}$ is the actual average ratio of insurance to value; call this ${}^{\overline{b}}\!/_{10}$.

Then $\overline{I} = N \, {}^{\overline{b}}\!/_{10} V.$

$$\overline{R} = \overline{L}/\overline{I} = \frac{NJV\overline{\lambda}}{N \, {}^{\overline{b}}\!/_{10} V} = J \, \overline{\lambda} \Big/ {}^{\overline{b}}\!/_{10}.$$

Let $\overline{\lambda} \Big/ {}^{\overline{b}}\!/_{10} = \overline{\rho};$ (12)

then $\overline{R} = J\overline{\rho}$ (11)

and $\overline{l} = JV\overline{\lambda} = {}^{\overline{b}}\!/_{10} V \overline{R}.$ (13)

$\overline{l}$ is in reality made up of 100 elementary expectations by (10), (9) and (2) but it reduces by (13) to an equivalent single expectation in which the amount at stake is the actual average insurance ${}^{\overline{b}}\!/_{10} V$ and the probability of this being called out is $\overline{R}$, the burning-ratio or ordinary net rate; $\overline{R}$ again is the product of the ignition hazard J and the damage hazard $\overline{\rho}$.

J is independent of the ν's but not so $\overline{\rho}$.

The number N is not known and therefore J cannot be found directly. The damage hazard $\overline{\rho}$ however can be computed and if the ordinary †rate $\overline{R}$ has already been ascertained J may be computed from the relation $\overline{R} = J\,\overline{\rho}$.

Since $R_x = J\,\rho_x$ and $\overline{R} = J\,\overline{\rho}$, $R_x = \overline{R}\cdot {\rho_x}/{\overline{\rho}}$. This gives the coinsurance-rates R_x in terms of the theoretically correct single rate $\overline{R}$. Let us consider however instead of $\overline{R}$ the rate actually in use $\widehat{R}$ which may or may not be correct. Let then $\widehat{R_x} = \widehat{R}\,{\rho_x}/{\overline{\rho}}$. These rates $\widehat{R_x}$ will produce exactly the same income as the single rate $\widehat{R}$, for:

$$\text{the income is } \sum_0^9 n_i\, {b_i}/{10}\; V\,\widehat{R'}$$

$$= \sum_0^9 n_i\, {b_i}/{10}\; V\,\widehat{R}\,{\rho'_i}/{\overline{\rho}}$$

$$= \frac{N\,V\,\widehat{R}}{\overline{\rho}} \sum_0^9 \nu_i\, {b_i}/{10}\; \rho'_i$$

$$= \frac{N\,V\,\widehat{R}\,\overline{\lambda}}{\overline{\rho}} = N\,{\overline{b}}/{10}\; V\,\widehat{R} = \overline{I}\,\widehat{R}.$$

If $\widehat{R} = \overline{R}$, this income is the expected loss $\overline{L}$.

This equivalence in the results of using $\widehat{R_x}$ and $\widehat{R}$ is conditional however upon the ν's remaining the same. The rates $\widehat{R_x}$ will produce an income equal to ${\widehat{R}}/{\overline{R}}$ times the expected loss whatever the value of the ν's; the rate $\widehat{R}$ however will produce an income equal to $\frac{\widehat{R}}{\overline{R^*}}$ times the expected loss where $\overline{R^*}$ is determined from a new set of numbers, ν^*, just as $\overline{R}$ is determined from the ν's. The second multiplier is a function of the ν^*'s while the first is not; the two will agree in general only if the ν^*'s are the same as the ν's.

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†Since $\overline{R}$ is independent of V it may be obtained in the ordinary way, without any assumption as to V by dividing the entire actual insurance-loss by the entire actual amount of insurance in force.

The quantities that it is important to determine are the damage-hazards ρ_x and $\overline{\rho}$. It is convenient to throw the work into tabular form by means of recursion formulas, and in this form some of the auxiliary quantities will be of interest.

The formulas are obtained thus:

$$L_x = V/_{10} \left\{ \sum_{0}^{x-1} a_i m_i + x \sum_{x}^{9} m_i \right\}.$$

$$L_{x+1} = V/_{10} \left\{ \sum_{0}^{x} a_i m_i + (x+1) \sum_{x}^{9} m_i \right\}.$$

$$L_{x+1} = L_x + \left\{ (a_x - x) m_x + \sum_{x+1}^{9} m_i \right\} V/_{10}.$$

$$\text{Let} \left\{ (a_x - x) m_x + \sum_{x+1}^{9} m_i \right\} V/_{10} = C_x;$$

$$\text{then } L_{x+1} = L_x + C_x.$$

$$\text{Let } \sum_{x+1}^{9} m_i = M_x, \text{ then } \sum_{x}^{9} m_i = M_{x-1}.$$

$$M_{x-1} = M_x + m_x.$$

Furthermore $M_9 = o$ and $L_0 = o$.

By these formulas the values L_x may be computed.

$$\lambda_x = \frac{L_x}{M\ V} \text{ and } \rho_x = \frac{\lambda_x}{x/_{10}},$$

$$\lambda'_x = \frac{\lambda_x + \lambda_{x+1}}{2},$$

$$\overline{\lambda} = \sum_{0}^{9} \nu_i \lambda'_i,$$

$$\overline{\rho} = \frac{\overline{\lambda}}{\overline{b}/_{10}}.$$

For actual computation we may conveniently take V to be one hundred.

The formulas then are:

$$M_9 = o.$$

$$M_{x-1} = M_x + m_x.$$

$$C_x = 10\left\{(a_x - x)\, m_x + M_x\right\}.$$

$$L_0 = o.$$

$$L_{x+1} = L_x + C_x.$$

$$\lambda_x = \frac{L_x}{100\,M}.$$

$$\rho_x = \frac{\lambda_x}{x/10}.$$

$$\lambda'_x = \frac{\lambda_x + \lambda_{x+1}}{2}.$$

$$\overline{\lambda} = \sum_0^9 \nu_i\, \lambda'_i.$$

$$\overline{\rho} = \frac{\overline{\lambda}}{\overline{b}/10}.$$

The work may be arranged in a table as follows, using for an example the figures for the class of frame mercantile buildings:

TABLE 21.—THE COMPUTATION FOR THE CLASS OF FRAME BUSINESS BUILDINGS.

x	0	1	2	3	4	5	6	7	8	9	10
a_x	.18	1.42	2.45	3.47	4.48	5.49	6.5	7.5	8.5	9.95	......
$a_x - x$	.18	.42	.45	.47	.48	.49	.50	.50	.50	.95	......
m_x	8293	576	326	215	139	97	69	49	42	194	......
M_x	1707	1131	805	590	451	354	285	236	194	0	......
$m_x (a_x - x)$	1492.74	241.92	146.70	101.05	66.72	47.53	34.50	24.50	21	184.30	......
$m_x(a_x - x) + M_x$	3199.74	1372.92	951.70	691.05	517.72	401.53	319.50	260 50	215 00	184.30	......
C_x	31997.40	13729.20	9517.00	6910.50	5177.20	4015.30	3195.00	2605.00	2150.00	1843.00	......
L_x	0	31997.40	45726.60	55243.60	62154.10	67331 30	71346.60	74541.60	77146.60	79296.60	81139.60
λ_x	0	.03200	.04573	.05524	.06215	.06733	.07135	.07454	.07715	.07930	.08114
ρ_x		.3200	.2286	.1841	.1554	.1347	.1189	.1065	.0964	.0881	.0811

In order to compute $\bar{\rho}$ it is convenient to throw the work into columns instead of rows, as follows:

TABLE 22.—THE COMPUTATION FOR THE CLASS OF FRAME BUSINESS BUILDINGS.

x	λ_x	$\lambda_x+\lambda_{x+1}$	λ'_x	ν_x	$\nu_x \lambda'_x$
0	———	.03200	.01600	0	0
1	.03200	.07773	.03886	0	0
2	.04573	.10097	.05048	$\frac{2}{127}$	.000795
3	.05524	.11739	.05869	$\frac{5}{127}$	.002311
4	.06215	.12948	.06474	$\frac{14}{127}$	.007137
5	.06733	.13868	.06934	$\frac{14}{127}$	.007644
6	.07135	.14589	.07295	$\frac{29}{127}$	.016656
7	.07454	.15169	.07584	$\frac{21}{127}$	.012540
8	.07715	.15645	.07822	$\frac{22}{127}$	.013550
9	.07930	.16044	.08022	$\frac{20}{127}$	.012633
10	.08114	———	———	———	———

$\bar{\lambda}$ = .073266

$\bar{b}/_{10}$, the actual average ratio of insurance to value, is found to be .7077

$$\bar{\rho} = \bar{\lambda} / \bar{b}/_{10} = .1035$$

By interpolation it is found that for x = 7.3, $\rho_x = \bar{\rho}$ that is in order for the coinsurance rates to produce the same income as the ordinary rate the 73 per cent coinsurance rate should equal the ordinary rate.

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